**The Possum Plague: Disease Dynamics**

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x' = c\*x - d\*y // total Possums

y' = y\*(x - y) -r\*y // Infected Possums

These equations model the dynamics of the spread of tuberculosis in the New Zealand possum population. The possum was introduced into New Zealand in 1830 to stimulate the fur trade. There were no natural predators of the possum on the island, and they soon proliferated. There are now more than 70 million possums in New Zealand! Unfortunately, possums can carry a form of tuberculosis (about half of them are infected) that can be communicated to livestock. Since cattle ranching is a very important part of the New Zealand's economy, there is a major effort underway to understand the ecology of the possum and the dynamics of the disease. The simplest of these models is treated here.

The equations given above are scaled versions of the original model which directly incorporates the biology. Those equations are:

P' = (a - b)P - uI

I' = vI(P - I) - (u + b)I

// P = total possum population

// I = infected possum population

where a and b are the natural intrinsic birth and death rates of possums, u is the disease-induced death rate constant, and v is the infectivity which measures the efficiency with which the population of infected possums (size I) communicates the disease to the population of healthy possums (size P - I). Once a possum catches the disease, it does not recover. Thus P is the size of the total possum population and P >= I.

For our purposes, think of x as a scaled version of P and y as a scaled version of I. The parameters c, d and r are, respectively, scaled versions of

1) c=(a-b)/v -intrinsic natural growth

2) d=u/v - disease induced attrition

3) r=(u+b)/v - combined attrition

For more information about this model, read the article by G. Wake in the Winter 1995 issue of the CODEE news- letter, or Section 5.5 in the textbook Differential Equations, A Modeling Perspective, by B. Borrelli and C. Coleman (1998, J. Wiley).

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**Project: find equilibria, study their stability and bifurcations under different conditions**

**1) Show that nontrivial equilibria (endemic infection level) exists only for c < 1**

**2) Fix d=1, find and describe the bifurcations of equilibria in terms of parameter c**

Try these combinations of parameters to see other kinds of behavior:

1. c = 0, r = 2

2. c = 1.5, r = 2

3. c = .25, r = 2

Added note by Jacob:

Maybe we should try when c = 0, r = 0. And then explain when both of these cases approach infinity.